The performance comparison of improved continuous mixed P-norm and other adaptive algorithms in sparse system identification

Afsaneh Akhbari[1], Aboozar Ghaffari[2]
Dept. of electrical engineering, Ghiaseddin Jamshid Kashani University, Abyek, Qazvin Province, Iran.
Dept. of electrical engineering, Sharif University of technology, Tehran, Iran.
[1] Afakhbari@gmail.com
[2]Aboozar412@gmail.com

***Abstract*— One of the important usages of adaptive filters is in sparse system identification on which the performance of classic adaptive filters is not acceptable. There are several algorithms that designed especially for sparse systems, we call them sparsity aware algorithms. In this paper we studied the performance of two newly presented adaptive algorithms in which P-norm constraint is considered in defining cost function. The general name of these algorithms is continuous mixed P-norm (CMPN). The performances of these algorithms are considered for the first time in sparse system identification. Also the performance of l_0 norm LMS algorithm is analyzed and compared with our proposed algorithms. The performance analyses are carried out with the steady-state mean square deviation (MSD) criterion of adaptive algorithms. We hope that this work will inspire researchers to look for other advanced algorithms against systems that are sparse.**

Keywords—Adaptive algorithms; sparse; mixed P-norm; system identification

I. Introduction

Sparse system identification is an important topic of research in the area of adaptive signal processing. Sparse systems are known to have long impulse responses with a few large taps and may be encountered in various applications including acoustic echo cancelation and channel estimation [1-10]. Especially in channel estimation application more and more adaptive algorithms are being presented because of the importance of this task in new generation mobile systems. Recently it was shown that the channel types for 5G and massive MIMO systems are sparse. For this reason, more sophisticated algorithms must be proposed for further improvement of sparse system identification. Up until now, the performance of so many algorithms are tested in sparse system identification. In [8] and [9] the performance of least mean forth (LMF) and its normalized versions are tested in converging to a sparse system. In [5] a variable step size normalized LMS algorithm was proposed for sparse system identification. The theoretical performance of these newly presented algorithms were investigated in a few papers like [11] but the theoretical performances of many sparsity aware algorithms are still not credited.

Recently a new type of adaptive algorithms are presented that are based on a continuous mixed P-norm (CMPN) [2]. Unlike other algorithms, the family of CMPN algorithms are rarely tested in the sparse system identification task and there is a plenty of work to be done in this topic. For example trying to improve and finding the optimum values for adjusting variables of these algorithms is a highly important topic for research. In recent years an improved version of this algorithm namely ICMPN algorithm was proposed but other improvements can be made for upgrading the performance of this algorithm in sparse system identification.

Our aim in this paper is to investigate the impact of various norms in performance of adaptive algorithms for sparse system identification. Our contribution is that for the first time we compared the performance of newly presented ICMPN algorithm with other sparsity aware algorithms. The l_0 norm algorithm is shown to be superior to all other presented algorithms when the system is highly sparse.

The rest of this paper is organized as follows:

In part II we briefly review sparse system identification. In part III we introduce some sparsity aware algorithms that are presented for sparse system identification then we will introduce the new improved mixed p-norm algorithm. In part IV we present our simulation results and compare them in sparse system identification and finally in part V we will present our concluding remarks and future scope.

II. Problem Statement

The sparse system is a system with a long impulse response that most of its elements are either zero or close to zero. In Fig. 1 we can see a sparse system response with 20 taps and only 4 non-zero elements. We show the system tap weight vector with $\boldsymbol{w}_o$ symbol.

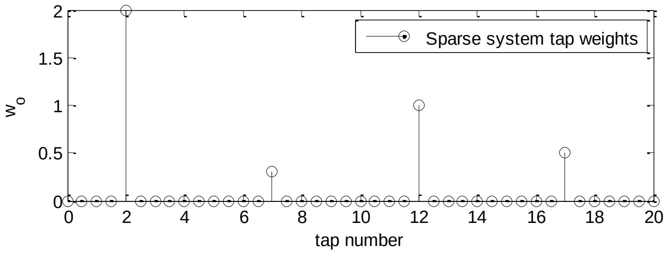

Fig. 1. Tap weights of a sparse system

For sparse system identification, we can use adaptive algorithms and try to converge their weights to taps of the system namely $\boldsymbol{w}_o$. In this case the system output can be written as:

$$y(n) = {\boldsymbol{w}_o}^T \boldsymbol{x}(n) + v(n) \qquad (1)$$

where $\boldsymbol{x}(n)$ is the input vector to the system and $v(n)$ is system noise. We begin with the classic LMS algorithm. If we consider $\boldsymbol{w}(n)$ to be the filter coefficient vector of LMS algorithm at iteration n, the weight vector update can be written as:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e(n)\boldsymbol{x}(n) \qquad (2)$$

Where μ is the step size and the convergence error is:

$$e(n) = y(n) - \boldsymbol{w}^T(n)\boldsymbol{x}(n) \qquad (3)$$

The MSD for this problem at iteration n can be calculated using:

$$MSD(n) = E\{\|\boldsymbol{w}_o - \boldsymbol{w}(n)\|^2\} \qquad (4)$$

The MSD criterion is the most suitable tool for investigating the performance of a newly proposed algorithm in sparse system identification.

III. SPARSE SYSTEM IDENTIFICATION ALGORITHMS

In this section we study one of the famous sparsity aware algorithms namely l_0 norm and then describe our newly presented algorithms.

A. *The l_0 norm adaptive algorithm*

The l_0 norm algorithm was proposed in [1] and it was shown that this algorithm has a reasonable performance in sparse system identification. The update relation of this algorithm can is given as:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e(n)x(n-i) - \kappa f_\beta(\boldsymbol{w}(n)) \qquad (5)$$

where $\kappa = \mu\gamma$ and $\gamma > 0$ is a regulating factor. The $f_\beta(.)$ function can be written as:

$$f_\beta(x) = \begin{cases} \beta^2 x + \beta & -\frac{1}{\beta} \le x < 0 \\ \beta^2 x - \beta & 0 < x \le \frac{1}{\beta} \\ 0 & elsewhere \end{cases} \qquad (6)$$

B. *The family of continuous mixed P-norm (CMPN) algorithms*

The (CMPN) algorithm was not originally proposed for sparse systems. But as it contains various norms in its nature, it can be adjusted to be used in sparse system identification. The update equation for this algorithm can be written as:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu\gamma_k \, sgn\,(e(k))\boldsymbol{x}(k) \qquad (7)$$

where the sign function is given as:

$$sgn(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad (8)$$

and for γ_k factor we have:

$$\gamma_k = \frac{(2|e(k)|-1)\ln(|e(k)|) - |e(k)|+1}{ln^2(|e(k)|)} \qquad (9)$$

Now we introduce the improved version of CMPN algorithm namely ICMPN. The updating equation of this algorithm is given as:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu\gamma_k \, sgn\,(e(k))\boldsymbol{x}(k)e^{-\beta|\boldsymbol{w}(k)|} \qquad (10)$$

where γ_k is defined in (9) and

IV. SIMULATION RESULTS

Here we present our simulation results for sparse system identification with the proposed algorithms. We consider a 256 tap system to identify. The system noise is considered to be white Gaussian with variance equal to 0.01. The inputs are also considered to be driven from a Gaussian distribution with unit variance. For all our simulations the step size factor is considered to be $\mu = 0.01$. The γ factor for l_0 norm algorithm is set to be 0.001 and as $\kappa = \mu\gamma$ the value of κ equals to 10^{-5}. Also for this algorithm the value of β is set to be 5. For the sale of comparison we run some simulations with different values of sparsity. As we can see from simulations the number of iterations that are needed for algorithms to converge are high because the number of system taps in all simulations are 256 and it represents a very long sequence.

A. *Sparse system identification*

In order to simulate a sparse system we consider a 256 tap system with only a few taps equal to 1 and set the others to be zero. In the first simulation, we assume only 28 taps are non-zero. We can see the steady state performance of proposed algorithms in Fig. 2.

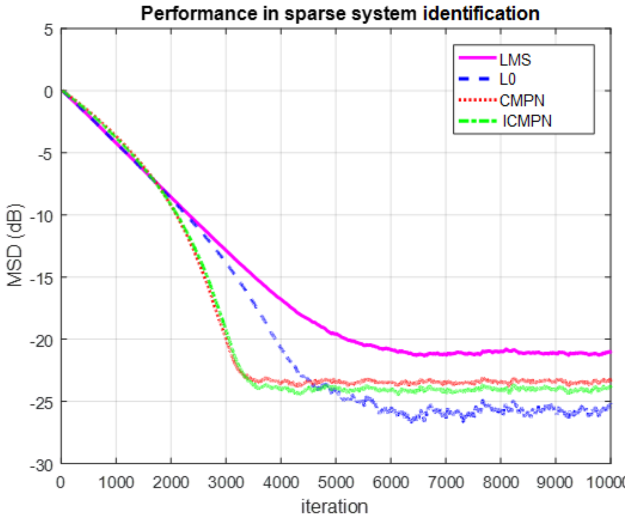

Fig. 2. The identification of a system with 256 taps and only 28 non-zero elements.

We can see that although the steady state performance of l_0 norm is better than ICMPN algorithm, the convergence speed of this algorithm is higher than l_0 norm. The convergence speed is a highly important factor in the performance of adaptive filters and it is presented by the number of iterations. Here we can see that the ICMPN algorithm converges nearly 1000 iterations faster than l_0 norm algorithm.

In the second simulation we raise the number of non-zero taps to 50. We can see in Fig. 3 that the steady state performance of l_0 norm algorithm derives close to the performance of ICMPN algorithm.

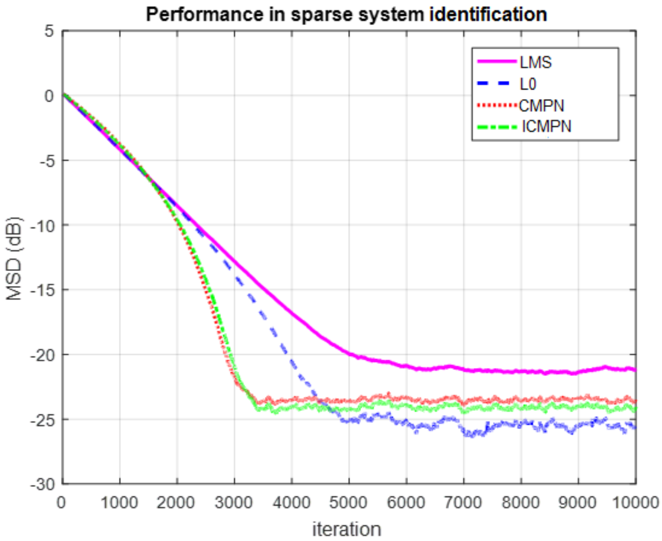

Fig. 3. The identification of a system with 256 taps and only 50 non-zero elements.

The performance of l_0 norm algorithm gets worse as the number of non-zero taps rise.

B. *Non-sparse system identificaion*

In order to make our system non-sparse we set the odd indexed taps to 1 while the remaining taps are set to -1. The performance of CMPN algorithm in the non-sparse system identification was claimed to be reasonable good [2]. In this simulation we show that both CMPN and ICMPN algorithms are originally presented for non-sparse systems. In Fig. 4. We can see that the performance of both algorithm are better than l_0 norm algorithm.

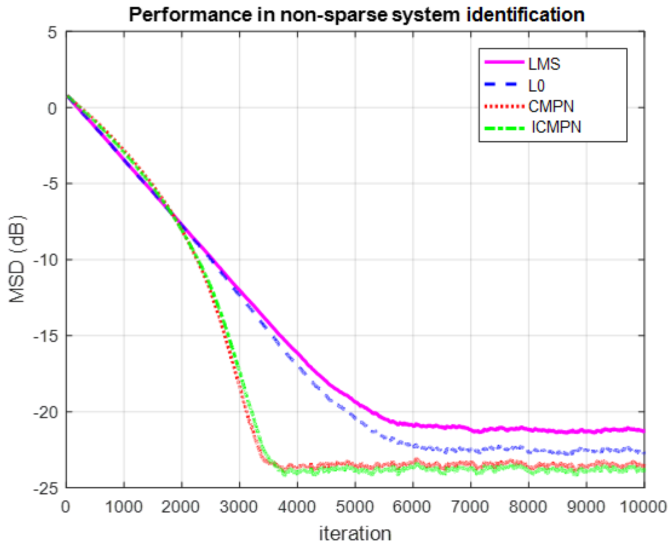

Fig. 4. The identification of a system with 256 non-zero taps.

The convergence speed of CMPN and ICMPN algorithms are higher than l_0 norm and LMS algorithms in both sparse and non-sparse conditions and this is due to the variable values of γ_k factor which is a kind of step size. Also the steady state performance of ICMPN algorithm in all cases is slightly better than that of CMPN algorithm.

V. COCLUSION

In this paper we compared the performance of the ICMPN algorithm with other sparsity aware algorithms in order to identify the weaknesses of this newly presented family of algorithms. While for non-sparse system identification the performance of ICMPN algorithm is better than others, in sparse system identification, the l_0 norm algorithm is shown to be superior to all other presented algorithms including ICMPN. Also we showed that the convergence speed of CMPN family is higher than others. This feature of CMPN algorithms can be used in the applications were identification speed is important. In future works we will work on the further improvements of

ICMPN algorithm and finding theoretical results for steady state performance of it for sparse system identification.

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